FEA Using the Anti-Symmetry Formulation of ECE EM Theory

Note 1: Equation System

The Equations

This discussion will look at the modeling equations for a conductive material embedded in a general three dimensional non-conductive region.

From equations (149) through (151) of "The Antisymmetry Law of Cartan Geometry: Applications to Electromagnetism and Gravitation"[1] we have

$$\omega_0 \underline{A} = -\frac{\partial}{\partial t}(\underline{\nabla}\,\psi) \qquad (149)$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(\underline{A} - \underline{\nabla}\psi) = \frac{1}{2}\,\mu_0\,\underline{J} + \frac{1}{2\varepsilon_0}\int \underline{\nabla}\rho\, dt \qquad (150)$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(\underline{\nabla}\,\Phi - \underline{\omega}\,\Phi) = \frac{1}{2}\,\mu_0\frac{\partial \underline{J}}{\partial t} + \frac{1}{2\varepsilon_0}\underline{\nabla}\rho \qquad (151)$$

The first thing that is immediately apparent is that we cannot have

$$\varphi = 0$$

unless we are prepared to have a relationship between current and charge that looks like

$$\underline{\nabla}\rho + \frac{1}{c^2}\frac{\partial \underline{\mathbf{J}}}{\partial t} = 0\,. \qquad (1)$$

This forces the current density to be static if the charge density is zero. Similar comments can be made about $\underline{\mathbf{A}}$ using equation (150).

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[1] The Antisymmetry Law of Cartan Geometry: Applications to Electromagnetism and Gravitation
M. W. Evans, D. W. Lindstrom, and H. Eckardt; A.I.A.S / T.G.A.; Paper 134

We note in passing that equation (1) when applied to equation (151) and to the combination of (149) and (150) also indicate that

$$\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi = 0$$
$$\frac{\partial \underline{\mathbf{A}}}{\partial t} + \omega_0 \underline{\mathbf{A}} = 0$$

These are the equations for the background radiation[2].

We will therefore insist that

$$\varphi \neq 0$$
$$\underline{\mathbf{A}} \neq 0$$

identically except in specific regions where the solution variable become zero.

We will however set

$$\rho = 0 .$$

Equations (149) through (151) become

$$-\frac{\partial}{\partial t}\left(\underline{\nabla}\psi\right) = \omega_0 \underline{\mathbf{A}} \tag{2}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) = \frac{1}{2}\mu_0 \underline{\mathbf{J}} \tag{3}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) = \frac{1}{2}\mu_0 \frac{\partial \underline{\mathbf{J}}}{\partial t} \tag{4}$$

This system consists of nine equations in nine unknowns, so appears to be completely specified. ψ is an unspecified scalar that is a result of Faraday's Law .[1] The meaning of ψ is unclear at this point.

Since we are dealing with a conductive material, everywhere within the conductor we assume that a linear constitutive relationship (Ohm's Law) between $\underline{\mathbf{J}}$ and $\underline{\mathbf{E}}$ is valid. For an isotropic material this is

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[2] Solution of the ECE Vacuum Equations
H. Eckardt and, D. W. Lindstrom; A.I.A.S / T.G.A.; Paper 134

$$\underline{\mathbf{J}} = -2\sigma \frac{\partial}{\partial t}\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) = -2\sigma\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) \tag{5}$$

This is a result of equations (124) and (125) of the above reference[1]. σ is the conductivity of the conductive material which may be zero outside of the material.

Substituting (5) into (3) and (4), we have

$$-\frac{\partial}{\partial t}\left(\underline{\nabla}\psi\right) = \omega_0 \underline{\mathbf{A}} \tag{6}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) + \sigma\mu_0 \frac{\partial}{\partial t}\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) = 0 \tag{7}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) + \sigma\mu_0 \frac{\partial}{\partial t}\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) = 0 \tag{8}$$

Equations (6), (7) and (8) represent the fields within the conductor.

In non-conductive regions, the fields are given by

$$-\frac{\partial}{\partial t}\left(\underline{\nabla}\psi\right) = \omega_0 \underline{\mathbf{A}} \tag{9}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) = 0 \tag{10}$$

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) = 0 \tag{11}$$

Boundary Conditions

The conditions on the interface between a conductive and non-conductive region are given by applying the divergence theorem to Gauss's and Coulomb's Law. The divergence theorem for an arbitrary vector **F** states

$$\iiint_v \underline{\nabla} \bullet \underline{\mathbf{F}}\,dv = \iint_s \underline{\mathbf{F}} \bullet \underline{\mathbf{n}}\,ds \tag{12}$$

Gauss's Law and Coulomb's Law are

$$\underline{\nabla} \bullet \underline{\mathbf{E}} = 0 \tag{13}$$

$$\underline{\nabla} \bullet \underline{\mathbf{B}} = 0 \tag{14}$$

where

$$\underline{\mathbf{E}} = -2\frac{\partial}{\partial t}\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) = -2\left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) \tag{15}$$

and

$$\underline{\mathbf{B}} = \underline{\nabla} \times \underline{\mathbf{A}} - \underline{\boldsymbol{\omega}} \times \underline{\mathbf{A}} \tag{16}$$

Equation (16) is simplified enormously is one adopts the Lindstrom Constraint.[3] In this case, equation (16) becomes

$$\underline{\mathbf{B}} = 2\underline{\nabla} \times \underline{\mathbf{A}} = -2\underline{\boldsymbol{\omega}} \times \underline{\mathbf{A}} \tag{17}$$

Inserting (15), (16) and (17) into (13) and (14), and applying (12) gives

$$\iint_s \frac{\partial}{\partial t}\left(\underline{\mathbf{A}} - \underline{\nabla}\psi\right) \bullet \underline{\mathbf{n}}ds = \iint_S \left(\underline{\nabla}\phi - \underline{\boldsymbol{\omega}}\phi\right) \bullet \underline{\mathbf{n}}ds = 0 \tag{18}$$

and

$$\iint_s \left(\underline{\nabla} \times \underline{\mathbf{A}} - \underline{\boldsymbol{\omega}} \times \underline{\mathbf{A}}\right) \bullet \underline{\mathbf{n}}ds = 0 \tag{19}$$

or

$$\iint_S \left(\underline{\nabla} \times \underline{\mathbf{A}}\right) \bullet \underline{\mathbf{n}}ds = \iint_S \left(\underline{\boldsymbol{\omega}} \times \underline{\mathbf{A}}\right) \bullet \underline{\mathbf{n}}ds = 0 \tag{20}$$

if the Lindstrom Constraint is used.

Note that in FlexPDE, these conditions translate to the terminology

$$Natural\left(\underline{\mathbf{F}}\right) = 0 \tag{21}$$

where $\underline{\mathbf{F}}$ is given by $\underline{\mathbf{E}}$ and $\underline{\mathbf{B}}$ in equations (15), (16) and (17).

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[3] Antisymmetry Constraints in the ECE Engineering Model
M. W. Evans, H. Eckardt, and D. W. Lindstrom; A.I.A.S / T.G.A.; Paper 133

Should values for the field variables $\underline{\mathbf{A}}$, ϕ, $\underline{\boldsymbol{\omega}}$, ω_0 or any of the intensities $\underline{\mathbf{E}}$ or $\underline{\mathbf{B}}$ be known on the boundaries, these values apply. These can be represented by

$$\begin{aligned}
\underline{\mathbf{A}} &= \underline{\mathbf{A}_s} \\
\phi &= \phi_s \\
\underline{\boldsymbol{\omega}} &= \underline{\boldsymbol{\omega}_s} \\
\omega_0 &= \omega_{0s} \\
\underline{\mathbf{E}} &= \underline{\mathbf{E}}_s \\
\underline{\mathbf{B}} &= \underline{\mathbf{B}}_s
\end{aligned} \tag{22}$$

where the symbols with the subscript "s" are the values for the corresponding field variables on the boundary labeled "s".

Loading Conditions

From an experimental perspective, loading of the system is accomplished by specifying $\underline{\mathbf{J}}$, ϕ, $\underline{\mathbf{E}}$, and/or $\underline{\mathbf{B}}$ at various points in the region at some or all values of time.

This is represented by

$$\begin{aligned}
\underline{\mathbf{J}} &= \underline{\mathbf{J}_s}\left(\underline{\mathbf{r}_0}, t\right) \\
\phi &= \phi_s\left(\underline{\mathbf{r}_0}, t\right) \\
\underline{\mathbf{E}} &= \underline{\mathbf{E}}_s\left(\underline{\mathbf{r}_0}, t\right) \\
\underline{\mathbf{B}} &= \underline{\mathbf{B}}_s\left(\underline{\mathbf{r}_0}, t\right)
\end{aligned} \tag{23}$$

for some regions given by $\underline{\mathbf{r}_0}$, for some or all values of time.

Initial Conditions

The values of at least some of the field variables must be known for the starting time of the problem. This is represented by

$$\begin{aligned}
\underline{\mathbf{A}}(\underline{\mathbf{r}}, 0) &= \underline{\mathbf{A}_i} \\
\phi(\underline{\mathbf{r}}, 0) &= \phi_i \\
\underline{\boldsymbol{\omega}}(\underline{\mathbf{r}}, 0) &= \underline{\boldsymbol{\omega}_i} \\
\omega_0(\underline{\mathbf{r}}, 0) &= \omega_{0i} \\
\underline{\mathbf{E}}(\underline{\mathbf{r}}, 0) &= \underline{\mathbf{E}}_i \\
\underline{\mathbf{B}}(\underline{\mathbf{r}}, 0) &= \underline{\mathbf{B}}_i
\end{aligned} \tag{24}$$

over the region specified by $\underline{\mathbf{r}}$ at initial time t=0.